3D Numerical Investigation of Distorted Scale in Hydraulic Physical Model Experiments

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ABSTRACT

In some physical model experiments, it is necessary to use distorted models. It is difficult, however, to build in an optimal degree of distortion into the models to ensure a closer degree of similarity between the model and the prototype. Three-dimensional (3D) numerical simulation is used in this paper to model experiments with a straight channel under different distorted scales. The calculated results of stream-wise, lateral and vertical velocities and sediment concentration along the vertical direction are shown by comparing the deviations of the velocities and sediment concentration between the distorted model and a normal one. Generally the discrepancy between the distorted model and the normal in stream-wise velocity is acceptable, while in vertical and transverse directions, the velocity shows differences. Concerning sediment concentration and channel bed deformation, the effect of the distorted scale is mainly related to two different similarity criteria. The similarity ratio between the turbulence diffusion velocity and gravity settling velocity can reproduce better results of sediment concentration along the vertical direction. The better bed deformation results come, however, from the similarity ratio between the averaged flow velocity and gravity settling velocity.

ADDITIONAL INDEX WORDS: physical model experiment, distorted model, numerical simulation, three dimensional, sediment concentration

INTRODUCTION

The use of physical model experiments is still an important tool for hydraulic project research, especially in such key projects in China as the Three Gorges Project (TGP) and the Gezhouba Project. When using physical model experiments, there are usually two problems which arise with the model design. First, the available laboratory space is limited, so that places a constraint on the horizontal scale. As a result it cannot be too large. Second, it is necessary to eliminate a series of problems due to the shallowness of the water in the model, so the vertical scale cannot be too small. The solution to both these problems is to construct the model using a distorted scale, with the horizontal scale larger than the vertical scale. The greater the degree of distortion, the more pronounced is the departure from the geometric similarity (e.g., Zhang and Xie, 1993).

The distorted model, however, contains many conflicts in similarity requirements which can never be fulfilled simultaneously. Concerning river models, for example, the ratio between the inertia force and the gravity force, and the ratio between the inertia force and the turbulent force are more important and usually used in the experiment (e.g., Yalin, 1972; Chen et al., 2004). As a result, the distorted model is relatively reinforced in the vertical direction, inducing dissimilarities of vertical velocity and vertical distribution of longitudinal velocity. As for mountainous streams which are not shallow water, the distortion leads to dissimilarities in the vortex, the circulating flow, and particularly the secondary flow along the horizontal axis (e.g., Cui et al., 2006). This, of course, causes a deviation in sediment movement. Past studies mainly present qualitative analyses of the errors which result from the distorted model and lack quantitative analyses of how to determine the distorted scale. Moreover, in some generalized physical model experiments, many 3D flow and sediment transport phenomena cannot be captured. Some empirical parameters are often used in distorted scale and sometimes produce uncertain results, so that further study needs to be carried out to decide quantitatively how the distorted scale impacts experiment results (e.g., Uittemaal, 2005; Vollmers, 1986; Wu and Chou, 2003).

The mathematical model has become a strong tool in the quantitative study of distorted scale of physical model experiments as computer science has developed. 3D numerical simulations achieve a lot of research on reproducing flow and sediment transport in the laboratory and natural condition. Scholars have presented much theoretical work in this field and set up some...
This paper presents a representative case study of a laboratory experiment to address how to select the distorted scale. The experiment on a straight channel river bed performed by WANG and RIBBERINK (1986) is used. The calculations for the above experiments and distorted model experiments are resolved using FAST3D code (e.g., ZHU, 1992; WU et al., 2000; FANG and RODI, 2003). The deviations in velocities and sediment concentration between the distorted model and the normal one in the case study experiment are compared and analyzed. Meanwhile, the suspended load concentration profile and the sediment deposition distribution along the longitudinal direction are used to explain two kinds of settling velocity similarity scales for their effect on different sediment transport model selection.

### 3D MATHEMATICAL MODEL FOR FLOW AND SEDIMENT TRANSPORT

#### Equations for Hydrodynamics

\[
\frac{\partial u_i}{\partial t} + \frac{\partial (u_j u_i)}{\partial x_j} = F_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \tag{1}
\]

where \( u_i (i = 1, 2, 3) \) are the velocity components, \( F_i \) is external force per unit volume fluid, \( \rho \) is fluid density, \( p \) is pressure, \( \tau_{ij} \) are the turbulent stresses calculated with the \( k - \varepsilon \) turbulence model which assumes \( \tau_{ij} = \rho \nu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} k \) and defines the eddy viscosity \( \nu_t \) as \( \nu_t = c_{\mu} k^2 / \varepsilon \); \( k \) and \( \varepsilon \) are turbulent kinetic energy and its dissipation rate, respectively.

In the absence of wind shear stress, the net fluxes of horizontal momentum and turbulent kinetic energy are specified as zero at the free surface. The velocity normal to the surface is set to zero and the pressure to the atmospheric value, while the dissipation rate \( \varepsilon \) is suggested to determine following RODI (1993),

\[
\varepsilon^{1/3} = k^{3/2} / (0.43 h)
\]

where \( h \) is local water depth and \( z_a \) is water level.

At the rigid wall of the bed and bank, the non-slip condition is used for velocity. The resultant wall shear stress \( \tau_w \) is calculated with,

\[
\tau_w = -\lambda_w \bar{V}_p \tag{4}
\]

where, \( \lambda_w = \mu / z_p \) for \( z_p < 11.6 \), and \( \lambda_w = \rho \varepsilon^{1/4} k^{1/2} \kappa / \ln(Ez_p) \) for \( z_p \geq 11.6 \);

\[
z_p = \rho \varepsilon^{1/4} k^{1/2} z_p / \mu \; ; \; \text{The subscript} \; p \; \text{refers to the first control-volume center near the wall, } \bar{V}_p \; \text{is the related flow velocity vector, } z_p \; \text{is the normal distance from the wall; } E \; \text{is the roughness parameter; The near-wall values of turbulent energy } k \; \text{and the dissipation rate } \varepsilon \; \text{are given as,}
\]

\[
k_p = \frac{\left| \tau_w \right|}{\rho \varepsilon^{1/3}} \varepsilon_p = \frac{c_{\mu}^{3/4} k_p^{3/2}}{\kappa z_p} \tag{5}
\]

#### Equation for Suspended Load Transport

The convection-diffusion equation of suspended load is,

\[
\frac{\partial s}{\partial t} + \frac{\partial}{\partial x_j} [(u_j - \omega \delta_{j3}) s] = \frac{\partial}{\partial x_j} \left( \frac{v_t}{\sigma_s} \frac{\partial s}{\partial x_j} \right) \tag{6}
\]

where, \( s \) is local sediment concentration, \( \omega \) is setting velocity of the sediment, \( \delta_{j3} \) is Kronecker delta with \( j = 3 \) indicating the vertical direction, \( \sigma_s \) is turbulent Schmidt number relating the diffusivity of the sediment to the eddy viscosity \( \nu_t \).

The boundary condition for suspended sediment concentration at the water surface is,
\[
\frac{v_s}{\sigma_s} \frac{\partial s}{\partial z} + \omega s = 0
\]  
(7)

The mass balance equation for river bed material can be written as,
\[
(1 - \rho') \frac{\partial z_b}{\partial t} + D_b - E_b = 0
\]  
(8)

where \( \rho' \) is the porosity of the bed material, \( z_b \) is the bed level above a fixed datum, \( D_b \) is the deposition rate of suspended sediment at the bed and \( E_b \) is the rate of entrainment of bed material into suspension. Hence, the net exchange of sediment material between bed and suspension is \( D_b - E_b \).

Following van Rijn (1987) and Celik and Rodi (1988), the deposition rate is \( D_b = \omega s_b \) and the entrainment rate is assumed that \( E_b = \omega s_b \), where \( s_b \) is the equilibrium sediment concentration just above the saltation layer. The net sediment flux near the bed therefore reads,
\[
D_b - E_b = \omega (s_b - s_b) \]  
(9)

To solve numerically the convection-diffusion equation of suspended load movement, three similarity conditions of sediment are obtained,
\[
\frac{\lambda_h}{\lambda_t \lambda_o} = 1, \quad \frac{\lambda_v}{\lambda_o \lambda_t} = 1, \quad \frac{\lambda_w}{\lambda_o \lambda_t} = 1
\]  
(10)

where \( \lambda_i \) is the horizontal geometry scale, \( \lambda_v \) is vertical geometry scale, \( \lambda_w, \lambda_u \) and \( \lambda_v \) are velocity scale in the stream-wise, vertical and transverse directions, \( \lambda_Q \) is the discharge scale.

From the convection-diffusion equation of suspended load movement, three similarity conditions of sediment are obtained,
\[
\frac{\lambda_h}{\lambda_t \lambda_o} = 1, \quad \frac{\lambda_v}{\lambda_o \lambda_t} = 1, \quad \frac{\lambda_w}{\lambda_o \lambda_t} = 1
\]  
(11)

where \( \lambda_o \) is the suspended load settling velocity scale, \( \varepsilon_x, \varepsilon_z, \varepsilon_v \) are turbulent diffusion coefficients of sediment in the horizontal and the vertical coordinates.

Based on the above equations, then
\[
\lambda_o = \lambda_h \lambda_t \lambda_v \]  
(12)

In the applied laboratory physical model experiment, if the study case concerns on the distribution of suspended load, the upward sediment flux will be zero, \( E_b = 0 \).

SIMILARITY CRITERIA USED IN LABORATORY MODEL EXPERIMENT

Similarity Criteria

Based on the similarity criteria of distorted model experiment, the similarity equations of 3D flow are,
\[
\lambda_u = \lambda_w = (\lambda_h)^{1/2}, \quad \lambda_v = \frac{\lambda_h}{\lambda_t}, \quad \lambda_w = \lambda_v \lambda_h \lambda_u
\]  
(13)

Comparing equations (13) and (14), then, it is obvious that the distorted model results do not strictly match the suspended load transport if only the normal one is used. In the applied laboratory physical model experiment, if the study case concerns on the distribution of suspended
study on the distorted scale

sediment movement, \( \lambda_{\omega} = \left( \frac{\lambda_h}{\lambda_l} \right)^{1/2} \lambda_u \) will be used, while if the problem concerns river bed deformation, \( \lambda_{\omega} = \frac{\lambda_h}{\lambda_l} \lambda_u \) will be used.

Stokes' formula for settling velocity is used to derive the sediment diameter scale. Following the two suspended load movement similarity criteria, two kinds of sediment diameter scale are obtained respectively.

\[
\begin{align*}
\lambda_d &= \frac{\lambda_s^{3/4} \lambda_u^{1/2}}{\rho - \rho_s} \quad \text{when} \quad \lambda_{\omega} = \frac{\lambda_h}{\lambda_l} \lambda_u \\
\lambda_d &= \frac{\lambda_s^{1/2} \lambda_u^{1/2}}{\rho - \rho_s} \quad \text{when} \quad \lambda_{\omega} = \left( \frac{\lambda_h}{\lambda_l} \right)^{1/2} \lambda_u
\end{align*}
\]

(15)

where \( \rho_s \) is the sediment particle density.

The next key similarity criterion for suspended load movement is the so called “sediment-carrying capacity” (e.g., ZHANG and XIE, 1993; FANG and WANG, 2000), which means the ability of the flow to transport suspended sediment measured as a percentage of volume. When the similarity of sediment concentration and vertical velocity profiles are ensured, the sediment concentration scale is required to be equal to the sediment-carrying capacity scale \( \lambda_s = \lambda_{s*} \).

By using the formula of sediment-carrying capacity

\[
S_c = \frac{\rho_s}{g R \omega} \left( f - f_s \right) \frac{U^3}{8 C_1 \rho - \rho_s}
\]

(16)

resistance of the sediment-laden flow, \( U \) is the cross-sectional averaged velocity; \( g \) is the gravitational acceleration; \( R \) is the hydraulic radius; and \( C_1 \) is the coefficient, then,

\[
\lambda_s = \lambda_{s*} = \frac{1}{\lambda_{c1}} \frac{\lambda_s^{3/4} \lambda_u^{1/2}}{\rho - \rho_s} \lambda_f \frac{\lambda_u^3}{\lambda_h \lambda_l \lambda_{\omega}}
\]

Assumed that \( \lambda_{c1} = 1 \), \( \lambda_f = \lambda_f = \frac{\lambda_h}{\lambda_l} \),

\[
\lambda_R = \lambda_h \quad \text{then,}
\]

\[
\begin{align*}
\lambda_s &= \lambda_{s*} = \frac{\lambda_s}{\rho - \rho_s} \lambda_l \quad \text{when} \quad \lambda_{\omega} = \frac{\lambda_h}{\lambda_l} \lambda_u \\
\lambda_s &= \lambda_{s*} = \frac{\lambda_s}{\rho - \rho_s} \left( \frac{\lambda_h}{\lambda_l} \right)^{1/2} \lambda_u \quad \text{when} \quad \lambda_{\omega} = \left( \frac{\lambda_h}{\lambda_l} \right)^{1/2} \lambda_u
\end{align*}
\]

(17)

Boundary Conditions for the Calculation Domain

If we suppose that each study case has the same horizontal geometry, namely \( \lambda_l = 1 \), then we can define the distortion scale as \( \eta = \frac{\lambda_l}{\lambda_h} \) and then,

\[
\lambda_u = \lambda_{u*} = \left( \frac{1}{\eta} \right)^{1/2} \quad \text{and} \quad \lambda_v = \left( \frac{1}{\eta} \right)^{3/2}
\]

(18)

In a straight channel river bed with a rectangular cross-section, where \( Q = A U = b h U \), in which \( A \) is the channel cross-sectional area and \( b \) is the width of the channel, then the scale of discharge is,

\[
\lambda_Q = \lambda_{Q*} = \left( 1 / \eta \right)^{3/2}
\]

(19)

The velocity at the inlet boundary of the distorted model is,

\[
u_{\eta} = \eta^{1/2} u_{11} \quad \text{and} \quad \nu_{\eta} = \eta^{3/2} v_{11} \quad \text{and} \quad w_{\eta} = \eta^{1/2} w_{11}
\]

(20)

where \( u_{\eta}, \nu_{\eta}, w_{\eta} \) are velocities in the stream-wise, vertical and transverse directions in the distorted model.
with a distortion rate of $\eta$; $u_{1:1}$, $v_{1:1}$, and $w_{1:1}$ refer to the velocities without distortion.

The water depth in the distorted model can be obtained as,

$$h_{\eta} = \frac{h_{1:1}}{\lambda_{h}} = \eta h_{1:1} \quad (21)$$

where $h_{\eta}$ and $h_{1:1}$ are water depth in the distorted model and then without distortion.

The sediment settling velocity and suspended load concentration in the distorted model are,

$$\omega_{\eta} = \frac{\omega_{1:1}}{\lambda_{\omega}} \quad , \quad S_{\eta} = \frac{S_{1:1}}{\lambda_{s}} \quad (22)$$

where $\omega_{\eta}$ and $\omega_{1:1}$ represent the sand settling velocity in the distorted model and without distortion, $S_{\eta}$ and $S_{1:1}$ are sediment concentration in distorted model and without distortion.

Following Equations (15) and (17), then,

$$\begin{cases} \omega_{\eta} = \frac{\omega_{1:1}}{\lambda_{\omega}} = \omega_{1:1} = \eta \quad \text{when} \quad \omega_{1:1} = \lambda_{s} = \lambda_{s} \\ \omega_{1:1} = \frac{\omega_{1:1}}{\lambda_{\omega}} = \lambda_{1:1}^{1/2} = \eta^{1/2} \quad \text{when} \quad \omega_{1:1} = \frac{(\lambda_{s})^{1/2}}{\lambda_{s}} = \lambda_{s}^{1/2} \quad (23) \end{cases}$$

and

$$\begin{cases} S_{\eta} = \frac{S_{1:1}}{\lambda_{s}} = \frac{S_{1:1}}{\lambda_{s}} = \frac{\lambda_{1:1}^{1/2}}{\rho} \quad \text{when} \quad \lambda_{s} = \lambda_{s} = \lambda_{s} \\ \lambda_{1:1}^{1/2} = \frac{(\lambda_{s})^{1/2}}{\lambda_{s}} \rho \quad S_{1:1} \quad \text{when} \quad \lambda_{s} = \lambda_{s}^{1/2} \quad \lambda_{s} \\ S_{\eta} = \frac{S_{1:1}}{\lambda_{s}} = \frac{\lambda_{1:1}^{1/2}}{\rho} \quad \text{when} \quad \lambda_{s}^{1/2} = \left(\frac{(\lambda_{s})^{1/2}}{\lambda_{s}} \right)^{1/2} \quad \lambda_{s} \end{cases} \quad (24)$$

**CASE STUDY**

The experiment carried out by **WANG** and **RIBBERINK** (1986) is used to explain how to select the distorted scale for the physical model experiment. The configuration of the experiment is shown in Figure 1. The dimension of the channel is 30 meters long, 0.5 meter wide and 0.5 meter high. The flow and sediment are fed to the channel in a steady, horizontally uniform rate over a perforated bottom which is constructed with opening percentage = 33% and opening diameter = 0.005m on a slope of 0.00097. The perforated bottom can trap the sediment particles and make a zero upward sediment flux ($E_{b} = 0$). The mean velocity is $u = 0.56 m/s$ and the water depth 0.215m. The input sediment concentration is $S = 70.8kg/hour$ with water discharge $Q = 0.0601m^{3}/s$. The characteristic sediment material diameters can be represented as $d_{10} = 0.075mm$, $d_{50} = 0.095mm$, and $d_{90} = 0.105mm$. In the calculations, uniform sediment material was assumed and, following **van RIJN** (1986), the fall velocity was taken as $\omega = 0.0065m/s$. Calculations for this case under the same conditions have been reported by **LIN** and **FALCONER** (1996) and **WU** et al., (2000).
Study on the Distorted Scale

Figure 1 Experiment configuration of straight channel (WANG and RIBBERINK, 1986)

Figure 2 Comparison of calculated and measured vertical velocity profile
Figure 3 Comparison of calculated and measured vertical sediment concentration
Figure 2 and Figure 3 show the comparison of the calculation and measurement of the vertical profiles of velocity and sediment concentration in the center line at different cross sections. It can be seen that the calculations agree well with the measurement. As shown in Figure 4, the results of sediment deposition also agree well with the measurement and the silt thickness is decreasing quickly along the channel in the stream-wise direction with the decreasing in sediment concentration.

Comparison of different distorted scale for the experiments

Four distorted scales with $\eta = 1, 2, 5, 10$ are used to perform the calculations of flow and sediment movement. Based on the transformation of geometry scale, the dimensions of the distorted calculation domains for $\eta = 1, 2, 5, 10$ are that the channel length and width are the same, namely 30 meters long and 0.5 meter wide, but the depths of the water are 0.215, 0.1075, 0.043 and 0.0215 meter respectively.

(1) Results of Flow

Figure 5 shows the vertical profile of the stream-wise velocity in the different distorted scale at the center line ($x=b/2$) and near the wall station ($x=b/4$). From the Figure it can be seen that the deviation is increasing in company with the increase in the distorted scale. However, the velocity profiles at different cross-sections almost match together, and it indicates that the deviation effect of the distorted scale on the stream-wise velocity is small.

(2) Results of sediment concentration and bed deformation

Equations (23) and (24) provide two types of boundary conditions for simulation of sediment concentration and bed deformation. The similarity scale between turbulence diffusion and gravity settling velocity $\lambda_u^{\ast} = \left( \frac{\lambda_u}{\lambda_L} \right)^{1/2}$ is used, then the equations

$\omega_q = \eta^{3/2} \omega_{13}$ and $S_q = \frac{\lambda_u^{\ast}}{\lambda_L^{\ast}} \eta^{1/2} S_{13}$

are specified as input boundary conditions. These are the first method sediment transport similarity criteria.
Study on the Distorted Scale

Figure 6 Results of sediment concentration profile by the first method
Study on the Distorted Scale

Figure 6 shows the vertical sediment concentration profile at different cross-sections with four kinds of distorted scale. It can be seen that the error factor between the distorted model and the normal one enlarges with the increase in the distorted scale along the stream-wise direction. The error factor becomes greatest at the outlet at the distortion scale $\eta = 10$. The figure also shows that the sediment concentration near the water surface is less than the normal one and that the near river bed is larger than the normal one.

Figure 7 shows the silt thickness. The lateral coordinate axis represents distance along the center line along the stream-wise direction, and the longitudinal axis represents the silting thickness.

It is obvious that the distorted model results show a large deviation in bed deformation as the silting thickness is 2 or 3 times higher than the normal one.

Moreover, the similarity scale between the averaged flow velocity and gravity settling velocity $\lambda \rho = \frac{\lambda_k}{\lambda_u}$ is used, then the equations $\omega_\eta = \eta \omega_{\eta_3}$ and $S_\eta = \frac{\rho}{\lambda \rho_S} S_{\eta_3}$ are used as the input boundary conditions. This is the second method for representing sediment transport similarity criteria.

The comparison of the vertical sediment concentration profile using a different distorted scale based on the second method is shown in Figure 8. From the data predicted in the six cross sections, it can be seen that the trend of deviation is similar with the results calculated using the former method but the values are much bigger.

Figure 9 shows the silt thickness in the center line along the straight channel using a different distorted scale based on the second method. It can be seen that the silt thickness using a different distorted scale generally agrees well. Before $x=17m$, the thickness is increasing with the increasing distorted scale, while after $x=17m$, the thickness is decreasing with the increasing distorted scale. Not only the trend, but also the value of the silt thickness using the second method is different from that used in the first method. The distorted scale gives little effect to the silt thickness in this method and the error is less than 12%.

Comparing the results obtained by the two methods using different similarity criteria, it can be concluded that the effect of distortion on sediment concentration distribution and silting thickness exists for both methods. The sediment concentration deviation calculated using the first method is small only if the model range is not so long, while it is larger than the deviation using the second one. For silting thickness, the deviation obtained
Study on the Distorted Scale

by the first method is much larger than the one obtained using the second method.

Figure 8 Results of sediment concentration profile by the second method
CONCLUSION

This paper presents a study of the effects of using distorted scale 3D numerical simulation in an hydraulic physical model experiment. The experiment on a straight channel river bed uses a study performed by WANG and RIBBERINK (1986). The calculation results of flow velocity, sediment concentration profile and sediment deposition are used to address the issue of deviation in the distorted model experiment.

Generally, the error factor between the distorted and the normal model on the flow velocity as along the hydrodynamic axis and the velocity profile in the stream-wise direction is small. In the cross section and vertical directions, however, the distorted scale strongly affects the velocity field. The deviation of the flow field may cause an inconsistency in the sediment carrying capacity and the sediment concentration profile. The selection of the distorted scale shows that using a distorted scale value of less than 2 basically guarantees the similarity of the secondary flow between the model and the prototype.

The distorted physical model experiment produces two kinds of similarity criteria of sediment transport. The similarity ratio between the turbulence diffusion velocity and the gravity settling velocity provides better results in the vertical sediment concentration profile and can be used in experiments involving intake and outlet in power plants. The similarity ratio between the averaged velocity and gravity settling velocity can generate better measurement results of sediment deposition and can be used in experiments involving reservoir and river bed deformation.


